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Two Proofs of the BINOMIAL THEOREM, by the REV. SAMUEL VINCE, A. M. F. R. S. Plumian Professor of Astronomy and experimental Philosophy, in the University of Cambridge

Read May, 1810

WHEN n is a whole positive number, it is proved by common algebra, that
$(1+x)^n = 1 + nx + n . \frac{n-1}{2} x^2 + \text{- - -} n . \frac{n-1}{2} \text{- - - -} \frac{n-r+1}{r} x^r +$ &c. Now if this be not true when n is a fraction, let the general co-efficient be $C + n . \frac{n-1}{2} \text{- - - - - -} \frac{n-r+1}{r} x^r$. Then the quantity C must vanish when $n = 1, 2, \text{- - - - -} \infty$. Now as C is expressed in terms of n and given co-efficients, it must always be of the same form whatever n is, and, as it must vanish when $n = 1, 2, \text{- - - -} \infty$, it must be represented by $n \times \overline{n-1} \times \overline{n-2} \times \text{- - - -} \overline{n-\infty} = n \left(n^r - a n^{r-1} + \text{\&c.} \right)$ where r is infinite; this therefore must be the value of C. But when n is a fraction, this value of C becomes infinite, which it cannot be, and as no other value of C can enter in addition, but this, the gene-

ral value of the co-efficient of x^r can be no other than $\frac{n.\,n-1}{2} \dots \frac{n-r+1}{r}$.

IDEM ALITER.

Let n and s be indefinitely great, whole, positive numbers, so that $\frac{n}{s}$ may represent any fraction; then by common algebra

$(1+x)^{ns} = 1+nsx + ns.\frac{ns-1}{2}x^2 + \&c.\ (P^s)$

$(1+x)^n = 1+nx + n.\frac{n-1}{2}x^2 + \&c.\ (P)$.

Now it is proved in my Fluxions, that $(1+x)^{\frac{n}{s}}$ may be represented by a series of the form

$1 + ax + bx^2 + \&c.\ (P)^{\frac{1}{s}}$, where the *form* of the series in respect to x is the same as that of the above series; we have therefore only to consider what is the relation of the corresponding co-efficients. Now the series (P^s) and (P) are exactly of the *same* form in *every* respect, the factor ns in the former being represented by n in the latter. If therefore we perform the *same* operation on these two series, the results must have the *same* form, and whatever change may take place on ns in (P^s), the same must take place on n in (P). If therefore we extract the s Root of (P^s) and (P), the forms of the two series expressing the roots must be the same, and the roots be deduced by the same rule. Now the reduction of (P^s) to (P) is made by writing for the quantity ns in (P^s) that quantity divided by the root s to be extracted, or writing n for ns; the reduction is therefore made simply by the root s to be extracted, dividing ns by s, and writing the quotient for ns; hence we extract the s^{th} root of (P) by the same rule, that is, by writing for n in (P), n divided by s; hence

$$(1+x)^{\frac{n}{s}}=1+\frac{n}{s}x+\frac{n}{s}\cdot\frac{\frac{n}{s}-1}{2}x^2+\text{\&c.}$$

It matters not *how* the s^{th} root of the series of the form $1+ax+bx^2+$ &c. can be extracted, or whether we should have been able to accomplish it if we had not known that the series (P^s) and (p) are represented by $(1+x)^{ns}$ and $(1+x)^n$. By *whatever* process the s^{th} root of (P^s) is extracted, whether discoverable or not, by the same process the s^{th} root of (P) will be extracted. The Binomial Theorem shews the series (P) to be the s^{th} root of (P^s), which is all we want to ascertain.